Math 241: Calculus and Analytic Geometry II

Exam 1

October 14, 2015

NAME:

To receive full credit you must clearly show all work and justify your answers. No books, notes, or calculators are allowed during this exam. This is a 50 minute exam.

Question:	1	2	3	4	5	6	Total
Points:	10	30	10	10	10	0	70
Score:							

1. (10 points) Evaluate the following limit:

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}.$$

2. (30 points) Evaluate the following integrals:

   (a) $\displaystyle\int \frac{x^3}{\sqrt{1+x^2}}\,dx$ (b) $\displaystyle\int \frac{x}{x^2+2x+5}\,dx$ (c) $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}\,dx.$

3. (10 points) Show the derivative of $\tan^{-1}(x)$ is $\dfrac{1}{1+x^2}$. State the domain and range of $\tan^{-1}(x)$.

4. (10 points) Determine if the following integral converges or diverges. If it converges, evaluate the integral.

$$\int_0^\infty te^{-t}\,dt.$$

5. (10 points) Determine if the following integral converges or diverges.

$$\int_0^1 \frac{\sec^2(x)}{x\sqrt{x}}\,dx$$

6. (10 points (bonus)) The gamma function is defined by

$$\Gamma(z) := \int_0^\infty e^{-t} t^{z-1} \, dt.$$

(a) Find $\Gamma(2)$, $\Gamma(3)$, and $\Gamma(4)$.

(b) Generalize $\Gamma(n+1)$ in terms of n, where n is a positive integer. (Hint: look for a pattern in your work from part (a).)