Math 310: Introduction to Abstract Mathematics

Exam 1

March 1, 2017

NAME:

To receive full credit you must clearly show all work and justify your answers. No books, notes, phones, or calculators are allowed during this exam. This is a 50 minute exam.

Question:	1	2	3	4	5	6	Total
Points:	5	15	10	10	5	10	50
Score:							

1. (5 points) Let I be an index set for a collection of sets $\{A_i\}_{i\in I}$. State the definitions for

$$\bigcup_{i\in I} A_i \quad \text{and} \quad \bigcap_{i\in I} A_i.$$

2. Let $A = \{b, c, d\}$ and $B = \{a, b\}$.

   (a) (3 points) Determine $A \times B$.

   (b) (3 points) Determine $|A \cup B|$.

   (c) (3 points) Determine $(A \times B) \setminus (B \times B)$.

   (d) (3 points) Determine $\mathcal{P}(A) \cap \mathcal{P}(B)$.

   (e) (3 points) Determine $|\mathcal{P}(A \times B)|$.

3. Let $S = \{a, b, c\}$. Consider the following open sentences with domain $D = \mathcal{P}(S)$.

$$P(A) : A \cap \{a, c\} \neq \emptyset \text{ and } Q(A) : A = \emptyset.$$

(a) (5 points) Determine all $A \in D$ such that $P(A) \Rightarrow Q(A)$ is false.

(b) (5 points) State the negation of the statement $\exists\, A \in D, P(A) \wedge (\sim Q(A))$ using symbols and determine if the negated statement is true or false.

4. Let P, Q, and R be statements.

   (a) (6 points) Let $X = (P \Rightarrow Q) \vee R$ and $Y =\sim (P \wedge (\sim Q)) \wedge (\sim R)$ Complete the following truth table.

P	Q	R	$\sim Q$	$\sim R$	$P \Rightarrow Q$	X	$P \wedge (\sim Q)$	$(P \wedge (\sim Q)) \wedge (\sim R)$	Y
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

   (b) (4 points) Is X logically equivalent to Y?

5. (5 points) Let A, B, and C be sets such that $A \cap B \cap C \neq \emptyset$. Draw a Venn Diagram that describes the set $(A \setminus C) \cup B$.

6. Let $C_0 = [0, 1]$. For $n \in \mathbb{N}$, define $C_n = C_{n-1} \setminus S_n$ where $S_n = \displaystyle\bigcup_{k=0}^{3^{n-1}-1} \left(\frac{1+3k}{3^n}, \frac{2+3k}{3^n} \right)$.

   (a) (3 points) Write C_1 as the union of two closed intervals.

   (b) (3 points) Write C_2 as the union of four closed intervals. (Hint: Draw C_2 on $[0, 1]$)

   (c) (2 points) Is $\displaystyle\bigcap_{n=0}^{\infty} C_n = \emptyset$?

   (d) (2 points) Find $\displaystyle\bigcup_{n=0}^{\infty} C_n$.