Math 310: Introduction to Abstract Mathematics

Exam 2

April 19, 2017

NAME:

To receive full credit you must clearly show all work and justify your answers. No books, notes, or calculators are allowed during this exam. This is a 50 minute exam.

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	10	0	50
Score:							

1. (a) (5 points) Let P and Q be statements. If you are proving $P \Rightarrow Q$ using proof by contradiction, what assumptions need to be made?

   (b) (5 points) Consider the statement: If a is an even integer and b is an odd integer, then $4 \nmid (a^2 + 2b^2)$. If you are proving this by contradiction, what assumptions do you make? (Note: you do **not** need to prove the statement.)

2. (a) (5 points) Give a counterexample to: for all $x, y \in \mathbb{R} \setminus \{0\}$, $4x^2 - 12xy + 9y^2 = 0$.

   (b) (5 points) Prove that there exists $x, y \in \mathbb{R} \setminus \{0\}$ such that $4x^2 - 12xy + 9y^2 = 0$.

3. (10 points) Let $x \in \mathbb{Z}$. Prove that x is even if and only if x^2 is even.

4. (10 points) Prove that $\sqrt{6}$ is an irrational number.

5. (10 points) The Fibonacci sequence $\{F_n\}_{n\in\mathbb{N}}$ is defined by $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, and for all $n \geq 2$ $F_n = F_{n-1} + F_{n-2}$. Use induction to prove that for all $n \in \mathbb{N}$ and for all $x \in \mathbb{R}$ such that $x^2 = x + 1$,

$$x^n = xF_n + F_{n-1}.$$

6. (5 points (bonus)) Prove that there exists $x, y \in \mathbb{R} \setminus \mathbb{Q}$ such that $x + y \in \mathbb{R} \setminus \mathbb{Q}$.