Introduction to Abstract Mathematics
MATH 310
Homework 9
Due Friday April 14

Please show all your work and justify your answers.

Exercise 1. Prove that $(1 + 2 + \cdots + n)^2 = 1^3 + 2^3 + \cdots + n^3$ for all $n \in \mathbb{N}$.

Exercise 2. Prove that $7 \mid (3^{2n} - 2^n)$ for every nonnegative integer n.

Exercise 3. Use the method of minimum counterexample to prove that $3 \mid (2^{2n} - 1)$ for every positive integer n.

Exercise 4. Prove that $5 \mid (n^5 - n)$ for every integer n.

Exercise 5. Consider the sequence $F_1, F_2, F_3, \ldots,$ where

$$F_1 = 1,\ F_2 = 1,\ F_3 = 2,\ F_4 = 3,\ F_5 = 5,\ \text{and } F_6 = 8.$$

The terms of this sequence are called **Fibonacci numbers**.

(a) Define the sequence of Fibonacci numbers by means of a recurrence relation.
(b) Prove that $2 \mid F_n$ if and only if $3 \mid n$.

Exercise 6. A sequence $\{a_n\}$ is defined recursively by $a_1 = 1$, $a_2 = 2$ and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Conjecture a formula for a_n and verify that your conjecture is correct.