Introduction to Abstract Mathematics
MATH 310
Homework 8
Due Friday April 7

Please show all your work and justify your answers.

Exercise 1. Show that there exists a rational number a and an irrational number b such that a^b is rational.

Exercise 2. Prove that there exist four distinct positive integers such that each integer divides the sum of the remaining integers.

Exercise 3. Disprove the statement: There is a real number x such that $x^6 + x^4 + 1 = 2x^2$.

Exercise 4. Use mathematical induction to prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for every positive integer n.

Exercise 5. Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$ for every positive integer n.

Exercise 6. Prove that if $A_1, A_2, \ldots, A_n$ are any $n \geq 2$ sets, then

$$\overline{A_1 \cap A_2 \cap \cdots \cap A_n} = \overline{A}_1 \cup \overline{A}_2 \cup \cdots \cup \overline{A}_n.$$