

Please show all your work and justify your answers.

Exercise 1. Prove that if $a, b \in \mathbb{Z}$, then $a^2 - 4b - 2 \neq 0$.

Exercise 2. If A and B are sets, prove that $A \cap (B \setminus A) = \emptyset$.

Exercise 3. Prove that the product of an irrational number and a nonzero rational number is irrational.

Exercise 4. Prove that $\sqrt{3}$ is irrational. (Hint: first prove for an integer a that $3 \mid a^2$ if and only if $3 \mid a$.)

Exercise 5. Prove that there are infinitely many positive integers n such that \sqrt{n} is irrational. (Hint: consider $\sqrt{2k}$ for any positive integer k .)

Exercise 6. For every positive $x \in \mathbb{Q}$, prove there is a positive $y \in \mathbb{Q}$ for which $y < x$.

Exercise 7. If $n \in \mathbb{Z}$, prove that $4 \nmid (n^2 + 2)$.

Exercise 8. Prove the number $\log_2(3)$ is irrational.