

Please show all your work and justify your answers.

Exercise 1. Let $n \in \mathbb{Z}$. Prove the following statements.

- (a) If n is even, then for all $k \in \mathbb{Z}$, $n \pm 2k$ is even.
- (b) If n is odd, then for all $k \in \mathbb{Z}$, $n \pm 2k$ is odd.

Exercise 2. Prove that if x is an odd integer, then $9x + 5$ is an even integer.

Exercise 3. Prove that if a and c are odd integers, then $ab + bc$ is even for every integer b .

Exercise 4. Let $x \in \mathbb{Z}$. Prove that $3x + 1$ is even if and only if $5x - 2$ is odd.

Exercise 5. Let $n \in \mathbb{Z}$. Prove that $(n + 1)^2 - 1$ is even if and only if n is even.

Exercise 6. Prove that if $n \in \mathbb{Z}$, then $n^2 - 3n + 9$ is odd.

Exercise 7. Let $x, y \in \mathbb{Z}$. Prove that if xy is odd, then x and y are odd.

Exercise 8. Let $x \in \mathbb{Z}$. Prove the following: If $x^3 - 1$ is even, then x is odd.