

Please show all your work and justify your answers.

**Exercise 1.** For statements  $P$  and  $Q$ , show that  $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$  is a tautology. Then state

$$(P \wedge (P \Rightarrow Q)) \Rightarrow Q$$

in words. (This type of logical argument is called **modus ponens**.)

**Exercise 2.** For statements  $P$ ,  $Q$ , and  $R$ , show that  $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$  is a tautology. Then state this compound statement in words. (This type of logical argument is called **sylogism**.)

**Exercise 3.** For statements  $P$  and  $Q$ , show that  $(\sim Q) \Rightarrow (P \wedge (\sim P))$  and  $Q$  are logically equivalent.

**Exercise 4.** Verify the following De Morgan's Law:  $\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q)$ .

**Exercise 5.** Let  $P$  and  $Q$  be statements. Show that  $[(P \vee Q) \wedge \sim (P \wedge Q)] \equiv \sim (P \Leftrightarrow Q)$ .

**Exercise 6.** Let  $S$  denote the set of odd integers and let

$$P(x) : x^2 + 1 \text{ is even. and } Q(x) : x^2 \text{ is even.}$$

be open sentences over the domain  $S$ . State  $\forall x \in S, P(x)$  and  $\exists x \in S, Q(x)$  in words.

**Exercise 7.** For the following quantified statements, first write them using symbols, then state their negations.

- (a) For every rational number  $r$ , the number  $1/r$  is rational.
- (b) There exists a rational number  $r$  such that  $r^2 = 2$ .

**Exercise 8.** Consider the open sentence

$$P(x, y, z) : (x - 1)^2 + (y - 2)^2 + (z - 2)^2 > 0.$$

where the domain of each of the variables  $x, y, z$  is  $\mathbb{R}$ .

- (a) Express the quantified statement  $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}, P(x, y, z)$  in words.
- (b) Is the quantified statement in (a) true or false? Explain.
- (c) Express the negation of the quantified statement in (a) in symbols.
- (d) Express the negation of the quantified statement in (a) in words.
- (e) Is the negation of the quantified statement in (a) true or false? Explain.

**Exercise 9.** Consider the open sentence  $P(a, b) : a/b < 1$ . where the domain of  $a$  is  $A = \{2, 3, 5\}$  and the domain of  $b$  is  $B = \{2, 4, 6\}$ .

- (a) State the quantified statement  $\forall a \in A, \exists b \in B, P(a, b)$  in words.
- (b) Show the quantified statement in (a) is true.

**Exercise 10.** Consider the open sentence  $Q(a, b) : a - b < 0$ . where the domain of  $a$  is  $A = \{3, 5, 8\}$  and the domain of  $b$  is  $B = \{3, 6, 10\}$ .

- (a) State the quantified statement  $\exists b \in B, \forall a \in A, Q(a, b)$  in words.
- (b) Show the quantified statement in (a) is true.