Please show all your work and justify your answers.

Exercise 1. Prove or disprove: The union of two equivalence relations on a nonempty set is an equivalence relation.

Exercise 2. A relation R is defined on $\mathbb{Z}$ by $a\,R\,b$ if $2a + 2b \equiv 0 \pmod{4}$. Prove that R is an equivalence relation and determine the distinct equivalence classes.

Exercise 3. Let R be a relation defined on $\mathbb{Z}$ by $a\,R\,b$ if $a^2 \equiv b^2 \pmod{5}$. Prove that R is an equivalence relation and determine the distinct equivalence classes.

Exercise 4. Let $A = \{5, 6\}$, $B = \{5, 7, 8\}$, and $S = \{n \in \mathbb{Z} \mid n \geq 3 \text{ is odd}\}$. A relation R from $A \times B$ to S is defined as $(a, b)\,R\,s$ if $s \mid (a + b)$. Is R a function from $A \times B$ to S?

Exercise 5. For a function $f : A \to B$ and subsets C and D of A and E and F of B, prove the following.
(a) $f(C \cup D) = f(C) \cup f(D)$
(b) $f(C \cap D) \subseteq f(C) \cap f(D)$
(c) $f(C) \setminus f(D) \subseteq f(C \setminus D)$
(d) $f^{-1}(E \cup F) = f^{-1}(E) \cup f^{-1}(F)$
(e) $f^{-1}(E \cap F) = f^{-1}(E) \cap f^{-1}(F)$
(f) $f^{-1}(E \setminus F) = f^{-1}(E) \setminus f^{-1}(F)$.