

Please show all your work and justify your answers.

Exercise 1. For the relation $R = \{(x, y) \mid x \leq y\}$ defined on \mathbb{N} , what is R^{-1} ?

Exercise 2. A relation R is defined on \mathbb{Z} by $x R y$ if $x \cdot y \geq 0$. Prove or disprove the following:

- (a) R is reflexive.
- (b) R is symmetric.
- (c) R is transitive.

Exercise 3. (a) Let R be the relation defined on \mathbb{Z} by $a R b$ if $a + b$ is even. Prove that R is an equivalence relation and determine the distinct equivalence classes.

(b) Suppose that “even” is replaced by “odd” in (a). Which of the properties reflexive, symmetric, and transitive does R possess?

Exercise 4. Let $H = \{2^m \mid m \in \mathbb{Z}\}$. A relation on R is defined on the set \mathbb{Q}^+ of positive rational numbers by $a R b$ if $\frac{a}{b} \in H$.

- (a) Prove that R is an equivalence relation.
- (b) Describe the elements in the equivalence class $[3]$.

Exercise 5. A relation R on a nonempty set A is defined to be **circular** if whenever $x R y$ and $y R z$, then $z R x$ for all $x, y, z \in A$. Prove that a relation R on A is an equivalence relation if and only if R is circular and reflexive.

Exercise 6. Let R be a relation on the set \mathbb{N} by $a R b$ if either $a \mid 2b$ or $b \mid 2a$. Prove or disprove: R is an equivalence relation.