

Please show all your work and justify your answers.

Exercise 1. Which of the following are sets?

- (a) $1, 2, 3$
- (b) $\{1, 2\}, 3$
- (c) $\{\{1\}, 2\}, 3$
- (d) $\{1, \{2\}, 3\}$
- (e) $\{1, 2, a, b\}$.

Exercise 2. Write each of the following sets by listing their elements between braces.

- (a) $A = \{2x + 1 \mid x \in \mathbb{Z}\}$
- (b) $B = \{x \in \mathbb{N} \mid -2 < x \leq 7\}$
- (c) $C = \{x \in \mathbb{R} \mid \cos(x) = 1\}$.

Exercise 3. Determine the cardinality of each of the following sets:

- (a) $A = \{1, 2, 3, 4, 5\}$
- (b) $B = \{x \in \mathbb{N} \mid |x| < 10\}$
- (c) $C = \{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}$
- (d) $D = \{\{1, 2\}, \{1, 2, 3, 4\}\}$
- (e) $E = \{\emptyset, \{\emptyset\}\}$
- (f) $F = \{x \in \mathbb{N} \mid 5x \leq 20\}$.

Exercise 4. Write each set using set-builder notation.

- (a) $A = \{-1, -2, -3, \dots\}$
- (b) $B = \{-3, -2, \dots, 3\}$
- (c) $C = \{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots\}$.

Exercise 5. For $A = \{2, 3, 5, 7, 8, 10, 13\}$, let

$$B = \{x \in A \mid x = y + z, \text{ where } y, z \in A\} \text{ and } C = \{r \in B \mid r + s \in B \text{ for some } s \in B\}.$$

Determine C .

Exercise 6. For each of the following give examples of three sets A, B and C such that

- (a) $A \subseteq B \subset C$
- (b) $A \in B, B \in C$, and $A \notin C$
- (c) $A \in B$ and $A \subset C$.

Exercise 7. Which of the following sets are equal?

$$A = \{n \in \mathbb{Z} \mid |n| < 2\}, B = \{n \in \mathbb{Z} \mid n^3 = n\}, C = \{n \in \mathbb{Z} \mid n^2 \leq n\}, \\ D = \{n \in \mathbb{Z} \mid n^2 \leq 1\}, E = \{-1, 0, 1\}.$$

Exercise 8. Find $\mathcal{P}(A)$ and $|\mathcal{P}(A)|$ for $A = \{0, \emptyset, \{\emptyset\}\}$.

Exercise 9. Let $U = \{1, 3, \dots, 15\}$ be the universal set, $A = \{1, 5, 9, 13\}$, and $B = \{3, 9, 15\}$. Determine the following:

- (a) $A \cup B$
- (b) $A \cap B$
- (c) $A \setminus B$
- (d) $B \setminus A$
- (e) \overline{A}
- (f) $A \cap \overline{B}$.

Exercise 10. Give examples of three sets A, B , and C such that $B \neq C$ but $B \setminus A = C \setminus A$.