Math 112: Introduction to Contemporary Mathematics

Exam 2

April 14, 2016

NAME:

To receive full credit you must clearly show all work and justify your answers. No books, notes, calculators, or any other electronic devices are allowed during this exam. This is a 75 minute exam.

Question:	1	2	3	4	5	6	7	8	Total
Points:	15	15	10	10	15	15	10	0	90
Score:									

1. (15 points) Determine if the following statements are true or false. Justify your responses.

   (a) If a complete graph has 5 vertices, then it has 120 Hamilton circuits.

   (b) If $0 = x^2 - 4x - 2$, then $x = 2 \pm \sqrt{8}$

   (c) A connected graph can have exactly one odd vertex.

   (d) A tree with n edges has $n - 1$ vertices.

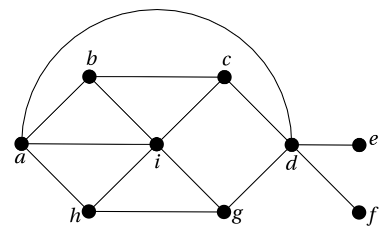
   (e) In a graph with 37 edges, the sum of all vertex degrees is 74.

2. Consider the graph below.

   (a) (5 points) Find the degree of **each** vertex and determine how many even vertices and how many odd vertices the graph has.

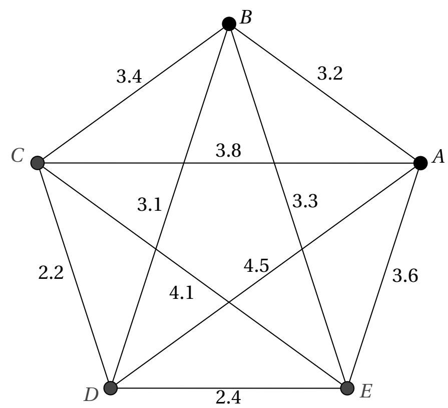
   (b) (5 points) Determine if the graph has an Euler circuit, Euler path, or neither. (Justify)

   (c) (5 points) If the graph has an Euler path, find one. If it does not, eulerize it so that it has an Euler path and find an Euler **path**.



3. Consider the weighted graph below.
   (a) (5 points) Find a tour using the nearest neighbor algorithm starting at vertex A.
   (b) (5 points) Find a tour using the cheapest link algorithm starting at vertex A.

   (Explain all your steps)



4. (10 points) Willie's Water Co. has to run a pipeline between the four towns Augusta (A), Bridgewater (B), Castle Rock (C), and Dustville (D). If the circuit $A-B-C-D-A$ forms a square with the distance along each edge of the circuit 10 miles, find the length of the shortest network between the four towns. (You may assume that you can construct pipe anywhere between the towns.) *Hint: In a 30-60-90 triangle the length of the shortest side is half the length of the hypotenuse.*

5. Consider the following table of tasks, processing times, and precedence relations.

Task	**Processing Time**	**Preceding Tasks**
A	5	C
B	5	C, D
C	5	
D	2	G
E	15	A, B
F	6	D
G	2	
H	2	G

(a) (5 points) Construct a digraph from the information given in the above table.

(b) (5 points) Use the backflow algorithm to find critical times for every task.

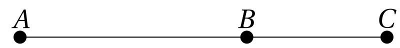
(c) (5 points) Find a schedule for the tasks using the critical-path-algorithm with two processors.

6. (a) (5 points) Use the recursive definition to calculate the first 10 Fibonacci numbers.

   (b) (5 points) If $F_7 F_x = F_9^2 - F_8^2$, what is x?

   (c) (5 points) If $F_n F_x = F_{n+2}^2 - F_{n+1}^2$, determine x. Verify your conclusion.

7. (10 points) Consider the line segment below where $\frac{AB}{BC} = \frac{AC}{AB}$. If $AB = x$ and $AC = y$, find the exact value of $\frac{AB}{BC}$. *Hint: You may use the fact that* $\frac{1}{\phi} = \frac{-1+\sqrt{5}}{2}$.



8. (9 points (bonus)) If $\alpha = \frac{1+\sqrt{5}}{2}$ and $\beta = \frac{-1+\sqrt{5}}{2}$, find

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\alpha}}}}} \quad \text{and} \quad \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\beta}}}}}.$$