Math 242: Calculus and Analytic Geometry III

Exam 3

April 12, 2016

NAME:

To receive full credit you must clearly show all work and justify your answers. No books, notes, or calculators are allowed during this exam. This is a 50 minute exam.

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	10	0	50
Score:							

1. (10 points) Evaluate the following double integral over the region $R = [-1, 1] \times [0, \frac{\pi}{2}]$:

$$\iint_R x \cos(y)\, dA.$$

2. (10 points) Evaluate the following double iterated integral. $\int_0^1 \int_0^{\sqrt{y}} \cos\left(3x - x^3\right)\, dx\, dy$.

3. (10 points) Find the volume of the solid that is inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

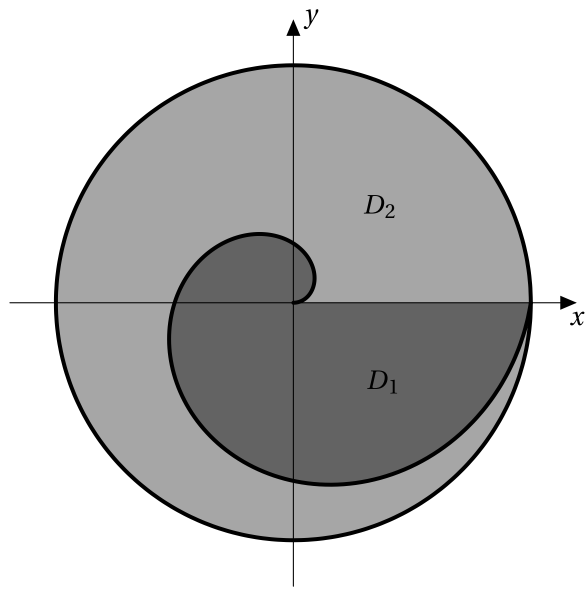
4. (10 points) Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane and below the cone $z = \sqrt{x^2 + y^2}$.

5. (10 points) Convert **but do not evaluate** the following triple iterated integral into a triple iterated integral in spherical coordinates: $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\,dz\,dy\,dx.$

6. (5 points (bonus)) Recall that the polar equation for a spiral is $r = a\theta$ for some number a. Archimedes gave the following proposition in his treatise on spirals:

   The area bounded by the first turn of the spiral and the initial line is equal to one-third of the circle bounding the first turn of the spiral. (In other words: $A(D_1) = \frac{1}{3}A(D_2)$ see the picture below.)

   Use a double integral to give a proof of Archimedes' proposition. [1]



[1] Archimedes proved this proposition using the method of exhaustion.