Math 242: Calculus and Analytic Geometry III

Exam 2

March 8, 2016

NAME:

To receive full credit you must clearly show all work and justify your answers. No books, notes, or calculators are allowed during this exam. This is a 50 minute exam.

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	10	0	50
Score:							

1. Consider the function $f(x,y) = \ln\left(\frac{x}{y}\right) + ye^{y^2+x^2}$.

   (a) (5 points) Find the gradient of $f(x,y)$.

   (b) (5 points) Find the directional derivative of $f(x,y)$ in the direction of the vector $\vec{v} = \langle 4,3\rangle$.

2. (10 points) Determine if the following limit exists. If it does show your answer using a $\delta - \varepsilon$ argument **or** squeeze theorem argument.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}.$$

3. (10 points) Find the point on the surface $y = x^2 + z^2$ such that the tangent plane is parallel to the plane $\frac{1}{2}x - y + \frac{3}{2}z = 2016$.

4. Consider the function $f(x, y) = x^4 + 2y^2 - 8xy$.
   (a) (5 points) Find all critical points of $f(x, y)$.
   (b) (5 points) Classify the critical points found in part (a).

5. Let $z = f(x, y)$ be any differentiable function with $x = s + t$ and $y = s - t$.

   (a) (5 points) Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

   (b) (5 points) Use part (a) to find $\left(\frac{\partial z}{\partial s}\right)\left(\frac{\partial z}{\partial t}\right)$ and show that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 - \left(\frac{\partial z}{\partial s}\right)\left(\frac{\partial z}{\partial t}\right) = 0.$$

6. (5 points (bonus)) Recall that the normal line to a surface at a point P is the line passing through P and perpendicular to the tangent plane at P.

   Show that every normal line to the sphere $(x-a)^2+(y-b)^2+(z-c)^2=r^2$ passes through the center of the sphere.