

Please show all your work and justify your answers.

Exercise 1. Give a counter example to the statement: Let $n \in \mathbb{N}$. If $\frac{n(n+1)}{2}$ is odd, then $\frac{(n+1)(n+2)}{2}$ is odd.

Exercise 2. Give a counterexample to the statement: For every positive real number x and every integer $n \geq 2$, the equation $x^n + (x+1)^n = (x+2)^n$ has no integer solutions.

Exercise 3. Prove that the product of an irrational number and a rational number is irrational.

Exercise 4. Prove that $\sqrt{3}$ is irrational. (Hint: first prove for an integer a that $3 \mid a^2$ if and only if $3 \mid a$.)

Exercise 5. Prove that there are infinitely many positive integers n such that \sqrt{n} is irrational. (Hint: consider $\sqrt{2k}$ for any positive integer k .)