

Please show all your work and justify your answers.

**Exercise 1.** Let  $n \in \mathbb{Z}$ . Prove the following statements.

- (a) If  $n$  is even, then for all  $k \in \mathbb{Z}$ ,  $n \pm 2k$  is even.
- (b) If  $n$  is odd, then for all  $k \in \mathbb{Z}$ ,  $n \pm 2k$  is odd.

**Exercise 2.** Prove that if  $x$  is an odd integer, then  $9x + 5$  is an even integer.

**Exercise 3.** Prove that if  $a$  and  $c$  are odd integers, then  $ab + bc$  is even for every integer  $b$ .

**Exercise 4.** Let  $x \in \mathbb{Z}$ . Prove that  $3x + 1$  is even if and only if  $5x - 2$  is odd.

**Exercise 5.** Let  $n \in \mathbb{Z}$ . Prove that  $(n + 1)^2 - 1$  is even if and only if  $n$  is even.

**Exercise 6.** Prove that if  $n \in \mathbb{Z}$ , then  $n^2 - 3n + 9$  is odd.

**Exercise 7.** Let  $x, y \in \mathbb{Z}$ . Prove that if  $xy$  is odd, then  $x$  and  $y$  are odd.

**Exercise 8.** Problem 3.40 in third edition of *Mathematical Proofs* or Problem 3.30 in the second edition of *Mathematical Proofs*.