Introduction to Abstract Mathematics
MATH 310
Homework 4
Due Friday 19 February

Please show all your work and justify your answers.

Exercise 1. For statements P and Q, show that $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ is a tautology. Then state

$$(P \wedge (P \Rightarrow Q)) \Rightarrow Q$$

in words. (This type of logical argument is called **modus ponens**.)

Exercise 2. For statements P, Q, and R, show that $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$ is a tatutology. Then state this compound statement in words. (This type of logical argument is called **syllogism**.)

Exercise 3. For statements P and Q, show that $(\sim Q) \Rightarrow (P \wedge (\sim P))$ and Q are logically equivalent.

Exercise 4. Verify the following De Morgan's Law: $\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q)$.

Exercise 5. Let P and Q be statements. Show that $[(P \vee Q) \wedge \sim (P \wedge Q)] \equiv \sim (P \Leftrightarrow Q)$.

Exercise 6. Let S denote the set of odd integers and let

$$P(x) : x^2 + 1 \text{ is even. and } \; Q(x) : x^2 \text{ is even.}$$

be open sentences over the domain S. State $\forall\, x \in S, P(x)$ and $\exists\, x \in S, Q(x)$ in words.

Exercise 7. For the following quantified statements, first write them using symbols, then state their negations.
(a) For every rational number r, the number $1/r$ is rational.
(b) There exists a rational number r such that $r^2 = 2$.

Exercise 8. Consider the open sentence

$$P(x, y, z) : (x - 1)^2 + (y - 2)^2 + (z - 2)^2 > 0.$$

where the domain of each of the variables x, y, z is $\mathbb{R}$ in words.
(a) Express the quantified statement $\forall\, x \in \mathbb{R}$, $\forall\, y \in \mathbb{R}$, $\forall\, z \in \mathbb{R}$, $P(x, y, z)$.
(b) Is the quantified statement in (a) true or false? Explain.
(c) Express the negation of the quantified statement in (a) in symbols..
(d) Express the negation of the quantified statement in (a) in words.
(e) Is the negation of the quantified statement in (a) true or false? Explain.

Exercise 9. Consider the open sentence $P(a, b) : a/b < 1$. where the domain of a is $A = \{2, 3, 5\}$ and the domain of b is $B = \{2, 4, 6\}$.
(a) State the quantified statement $\forall\, a \in A$, $\exists\, b \in B$, $P(a, b)$ in words.
(b) Show the quantified statement in (a) is true.

Exercise 10. Consider the open sentence $Q(a, b) : a - b < 0$. where the domain of a is $A = \{3, 5, 8\}$ and the domain of b is $B = \{3, 6, 10\}$.
(a) State the quantified statement $\exists\, b \in B$, $\forall \in A$, $Q(a, b)$ in words.
(b) Show the quantified statement in (a) is true.