Introduction to Abstract Mathematics MATH 310 Homework 3 Due Friday 12 February

Please show all your work and justify your answers.

Exercise 1. Give an example of a statement and determine it's truth value.

Exercise 2. Consider the open sentence P(x) : x(x-1) = 6 over the domain \mathbb{R} .

(a) For what values of x is P(x) a true statement?

(b) For what values of x is P(x) a false statement?

Exercise 3. For the open sentence $P(A) : A \subseteq \{1, 2, 3\}$ over the domain $S = \mathcal{P}(\{1, 2, 4\})$, determine:

- (a) all $A \in S$ for which P(A) is true.
- (b) all $A \in S$ for which P(A) is false.
- (c) all $A \in S$ for which $A \cap \{1, 2, 3\} = \emptyset$.

Exercise 4. State the negation of each of the following statements.

- (a) At least two of my library books are overdue.
- (b) One of my two friends misplaced his homework assignment.
- (c) No one expected that to happen.
- (d) It's not often that my instructor teaches that course.
- (e) It's not surprising that two students received the same exam score.

Exercise 5. For the sets $A = \{1, 2, ..., 10\}$ and $B = \{2, 4, 6, 9, 12, 25\}$, consider the statements

$$P: A \subseteq B. \ Q: |A \setminus B| = 6$$

Determine which of the following statements are true.

- (1) $P \lor Q$
- (2) $P \lor (\sim Q)$
- (3) $P \wedge Q$
- (4) $(\sim P) \land Q$
- (5) $(\sim P) \lor (\sim Q).$

Exercise 6. Let $S = \{1, 2, ..., 6\}$ and let $P(A) : A \cap \{2, 4, 6\} = \emptyset$. and $Q(A) : A \neq \emptyset$. be open sentences over the domain $\mathcal{P}(S)$.

- (1) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \wedge Q(A)$ is true.
- (2) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \lor (\sim Q(A))$ is true.
- (3) Determine all $A \in \mathcal{P}(S)$ for which $(\sim P(A)) \land (\sim Q(A))$ is true.

Exercise 7. For statements P and Q, construct a truth table for $(P \Rightarrow Q) \Rightarrow (\sim P)$.

Exercise 8. Let A and B be nonempty disjoint subsets of a set S. If $x \in S$, then which of the following statements are true?

- (1) It's possible that $x \in A \cap B$.
- (2) If x is an element of A, then x can't be an element of B.
- (3) If x is not an element of A, then x must be an element of B.
- (4) It's possible that $x \notin A$ and $x \notin B$.
- (5) For each nonempty set C, either $x \in A \cap C$ or $x \in B \cap C$.
- (6) For some nonempty set C, both $x \in A \cup C$ and $x \in B \cup C$.

Exercise 9. For open sentences P(x) : |x - 3| < 1. and $Q(x) : x \in (2, 4)$. over the domain \mathbb{R} , state the biconditional $P(x) \Leftrightarrow Q(x)$.

Exercise 10. For statements P and Q, the implication $(\sim P) \Rightarrow (\sim Q)$ is called the **inverse** of the implication $P \Rightarrow Q$.

- (1) Use a truth table to show that these statements are not logically equivalent.
- (2) Find another implication that is logically equivalent to $(\sim P) \Rightarrow (\sim Q)$ and verify your answer.