Introduction to Abstract Mathematics
MATH 310
Homework 11
Due Wednesday May 4

Please show all your work and justify your answers.

Exercise 1. Let $A = \{5, 6\}$, $B = \{5, 7, 8\}$, and $S = \{n \in \mathbb{Z} \mid n \geq 3 \text{ is odd}\}$. A relation R from $A \times B$ to S is defined as $(a, b)\, R\, s$ if $s \mid (a + b)$. Is R a function from $A \times B$ to S?

Exercise 2. For a function $f : A \to B$ and subsets C and D of A and E and F of B, prove the following.
(a) $f(C \cup D) = f(C) \cup f(D)$
(b) $f(C \cap D) \subseteq f(C) \cap f(D)$
(c) $f(C) \setminus f(D) \subseteq f(C \setminus D)$
(d) $f^{-1}(E \cup F) = f^{-1}(E) \cup f^{-1}(F)$
(e) $f^{-1}(E \cap F) = f^{-1}(E) \cap f^{-1}(F)$
(f) $f^{-1}(E \setminus F) = f^{-1}(E) \setminus f^{-1}(F)$.

Exercise 3. Give an example of two finite sets A and B and two functions $f : A \to B$ and $g : B \to A$ such that f is one-to-one but not onto and g is onto but not one-to-one.

Exercise 4. Let f be a function with $\text{dom}(f) = A$ and let C and D be subsets of A. Prove that if f is one-to-one, then $f(C \cap D) = f(C) \cap f(D)$.

Exercise 5. Let A and B be nonempty sets. Prove that if $f : A \to B$ is a function, then $f \circ i_A = f$ and $i_B \circ f = f$.

Exercise 6. Let A be a nonempty set and let $f : A \to A$ be a function. Prove that if $f \circ f = i_A$, then f is bijective.

Exercise 7. Let A, B and C be nonempty sets and let f, g and h be functions such that $f : A \to B$, $g : B \to C$ and $h : B \to C$. For each of the following, prove or disprove:
(a) If $g \circ f = h \circ f$, then $g = h$.
(b) If f is one-to-one and $g \circ f = h \circ f$, then $g = h$.

Exercise 8. Suppose, for a function $f : A \to B$, there exists a function $g : B \to A$ such that $f \circ g = i_B$. Prove that if g is surjective, then $g \circ f = i_A$.