# Math 310: Introduction to Abstract Mathematics

Exam 2

April 6, 2016

**NAME:**

To receive full credit you must clearly show all work and justify your answers. No books, notes, or calculators are allowed during this exam. This is a 50 minute exam.

| Question: | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Points: | 10 | 10 | 10 | 10 | 10 | 50 |
| Score: | | | | | | |

1. (a) (5 points) Let $P$ and $Q$ be statements. State the contrapositive of $P \Rightarrow Q$.

   (b) (5 points) State the contrapositive of the following: Let $A$ and $B$ be sets. If $x \in A \cap B$, then $x \in A$ and $x \in B$.

2. (a) (5 points) Give a counterexample to the statement: for all $x, y \in \mathbb{R}^{\geq 0}$, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

   (b) (5 points) Prove that there exists $x, y \in \mathbb{R}^{\geq 0}$ such that $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

3. (10 points) Let $a, b, c, n \in \mathbb{Z}$ such that $n \geq 2$. Prove that congruences are transitive. That is if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$.

4. (10 points) Prove that if $a, b \in \mathbb{Z}$ such that $a \geq 2$, then $a \nmid b$ or $a \nmid (b+1)$.

5. (10 points) The Fibonacci sequence $\{F_n\}_{n\in\mathbb{N}}$ is defined by $F_1 = 1$, $F_2 = 1$, and for all $n \geq 2$ $F_n = F_{n-1} + F_{n-2}$ (you may assume that $F_0 = 0$). Use induction to prove that for all $n \in \mathbb{N}$ and for all $x \in \mathbb{R}$ such that $x^2 = x + 1$,

$$x^n = xF_n + F_{n-1}.$$