Math 310: Introduction to Abstract Mathematics

Exam 1

February 24, 2016

NAME:

To receive full credit you must clearly show all work and justify your answers. No books, notes, or calculators are allowed during this exam. This is a 50 minute exam.

Question:	1	2	3	4	5	6	Total
Points:	5	10	15	10	5	10	50
Score:							

1. (5 points) Let U be a universal set. Given two sets A and B, describe $A \cap B$ and $\overline{A}$ using set notation (i.e., $\{x \mid p(x)\}$).

2. Let $A = \{\{\emptyset\}, \{1, 2\}\}$, $B = \{\emptyset, \{\emptyset\}\}$ and $C = \{\{\emptyset\}, \{2\}, \{\{1\}, 2\}\}$.

   (a) (2 points) Find $|A|$.

   (b) (2 points) Is $\emptyset \in A \cap C$?

   (c) (2 points) Find $|\mathcal{P}(C)|$.

   (d) (2 points) Find $B \cap C$.

   (e) (2 points) Is $B \subseteq C$?

3. Let $S = \{2, 3, 4\}$. Consider the following open sentences with domain $D = \mathcal{P}(S)$.

$$P(A) : A \cap \{2, 4\} = \emptyset \text{ and } Q(A) : A \neq \emptyset.$$

(a) (5 points) Find all $A \in D$ such that $P(A) \Rightarrow Q(A)$ is false.

(b) (5 points) Find all $A \in D$ such that $P(A) \Rightarrow Q(A)$ is true.

(c) (5 points) Find the negation of the statement $\exists\, A \in D, P(A) \wedge (\sim Q(A))$ and determine if the negated statement is true or false.

4. Let P, Q, and R be statements.

   (a) (6 points) Complete the following truth table.

P	Q	R	$\sim Q$	$\sim R$	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$	$P \wedge (\sim R)$	$(P \wedge (\sim R)) \Rightarrow (\sim Q)$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

   (b) (4 points) Is $(P \wedge Q) \Rightarrow R \equiv (P \wedge (\sim R)) \Rightarrow (\sim Q)$?

5. (5 points) Let A, B, and C be sets such that $A \cap B \cap C \neq \emptyset$. Draw a Venn Diagram that describes the set $(A \cup C) \setminus (A \cap B)$.

6. Let $C_0 = [0,1]$. For $n \in \mathbb{N}$, define $C_n = C_{n-1} \setminus S_n$ where $S_n = \displaystyle\bigcup_{k=0}^{3^{n-1}-1} \left(\frac{1+3k}{3^n}, \frac{2+3k}{3^n} \right)$.

   (a) (3 points) Write C_1 as the union of two closed intervals.

   (b) (3 points) Write C_2 as the union of four closed intervals. (Hint: Draw C_2 on $[0,1]$)

   (c) (2 points) Is $\displaystyle\bigcap_{n=0}^{\infty} C_n = \emptyset$?

   (d) (2 points) Find $\displaystyle\bigcup_{n=0}^{\infty} C_n$.