

Statistics
MATH 371
Homework 0
Part 2 of 2.

Exercise 1. Suppose that a point (X, Y) is chosen from the disk

$$S = \{(x, y) : (x - 1)^2 + (y + 2)^2 \leq 9\}.$$

Determine

- (a) the conditional pdf of Y for every given value of X .
- (b) $\Pr(Y > 0 | X = 2)$.

Exercise 2. Suppose that the pdf of X is

$$f(x) = \begin{cases} \frac{1}{2}x & \text{for } 0 < x < 2, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the pdf of $Y = 4 - X^2$.

Exercise 3. Suppose that one word is to be selected at random from the sentence "THE GIRL PUT ON HER BEAUTIFUL RED HAT." If X denotes the number of letters in the word that is selected, what is the value of $E(X)$?

Exercise 4. Suppose that X and Y have a continuous joint distribution for which the jpdf is

$$f(x, y) = \begin{cases} 12y^2 & \text{for } 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(XY)$.

Exercise 5. Suppose that three random variables X_1, X_2, X_3 form a random sample from a distribution for which the mean is 5. Determine the value of

$$E(2X_1 - 3X_2 + X_3 - 4).$$

Exercise 6. Let X be a random variable for which $E(X) = \mu$ and $\text{Var}(X) = \sigma^2$. Show that

$$E[X(X - 1)] = \mu(\mu - 1) + \sigma^2.$$

Exercise 7. Suppose that X and Y are independent random variables whose variances exist and such that $E(X) = E(Y)$. Show that

$$E[(X - Y)^2] = \text{Var}(X) + \text{Var}(Y).$$