Math 371: Statistics

Exam 2

November 23, 2016

NAME:

To receive full credit you must clearly show all work and justify your answers. No books or calculators are allowed during this exam. You are allowed one page (letter size) of notes. This is a 50 minute exam.

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	0	40
Score:						

1. (10 points) Let $X_1, \ldots, X_n$ be a random sample from a distribution with the following probability density function:

$$f(x|\theta) = \begin{cases} \left(\frac{1}{\theta}\right) e^{-x/\theta} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

Determine if $\overline{X}_n$ is an unbiased estimator of θ.

2. (10 points) Recall that the probability density function for a χ^2 distribution with m degrees of freedom is

$$f(x) = \frac{1}{2^{m/2}\Gamma\left(\frac{m}{2}\right)} x^{m/2-1} e^{-x/2}.$$

If X has a χ^2 distribution with m degrees of freedom, it is said that $X^{1/2}$ has a χ distribution with m degrees of freedom. Determine the mean of a χ distribution. (**Hint:** Calculate $E(X^{1/2})$ using the pdf from the χ^2 distribution.)

3. Consider a single observation X from a Cauchy distribution centered at θ. That is the pdf of X is

$$f(x|\theta) = \frac{1}{\pi[1 + (x - \theta)^2]}$$

for all $x \in \mathbb{R}$. Suppose that we wish to test the hypotheses

$$H_0 : \theta \leq \theta_0$$
$$H_1 : \theta > \theta_0.$$

Let δ_c be the test that rejects H_0 if $X \geq c$.

(a) (5 points) Show that $\pi(\theta|\delta_c)$ is an increasing function of θ.

(b) (5 points) Find c to make δ_c have size $\frac{1}{4}$.

4. (10 points) Prove that the method of moments estimator for the parameter of an exponential distribution is the maximum likelihood estimator. Recall the pdf for an exponential distribution:

$$f(x|\beta) = \begin{cases} \beta e^{-\beta x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

5. (10 points (bonus)) Assume that a random variable X has a t distribution with 1 degree of freedom. Prove that $\frac{1}{X}$ has a t distribution with 1 degree of freedom. (**Hint:** If a random variable Z has a Cauchy distribution, then Z can be expressed as $Z = \frac{U}{V}$ where U and V are independent random variables and each has a standard normal distribution.)