



1. (15 points) Determine if the following statements are true or false. Justify your responses.

(a) The plurality method satisfies the majority criterion.

(b) In an election with  $N$  candidates the total number of pairwise comparisons is  $\frac{(N-1)N}{2}$ .

(c) In a weighted voting system of  $n$  players, there are a total of  $2^n - 1$  possible **winning** coalition.

(d) In the weighted voting system  $[15 : 16, 8, 4, 1]$ , player 1 is a dictator.

(e) A pivotal player is a player that if they leave a winning coalition, then the coalition becomes a losing coalition.

2. Donnie (D), Jeffery (J), Quintana (Q), and Walter (W) have formed a bowling team. They decide to let the league bowlers elect their team captain. The results are listed in the preference schedule below.

Number of voters	12	7	13	3	8
First choice	J	Q	W	W	D
Second choice	D	J	D	D	Q
Third choice	Q	D	Q	Q	J
Fourth choice	W	W	J	J	W

- (a) (2 points) How many first place votes are needed for a candidate to have a majority?
- (b) (3 points) If the plurality method is used, who is the winner of the election? Does the plurality candidate have a majority?
- (c) (10 points) If the plurality with elimination method is used, who is the winner of the election?

3. Consider the following preference schedule from problem 2.

Number of voters	12	7	13	3	8
First choice	J	Q	W	W	D
Second choice	D	J	D	D	Q
Third choice	Q	D	Q	Q	J
Fourth choice	W	W	J	J	W

- (a) (10 points) Use the method of pairwise comparisons to determine the winning candidate.
- (b) (5 points) What fairness criterion does this example show that the plurality method violates? Justify your answer.

4. Consider a weighted voting system  $[q : w_1, w_2, w_3, w_4]$  of four players  $P_1, P_2, P_3,$  and  $P_4$ .
- (a) (5 points) What do  $q, w_1, w_2, w_3,$  and  $w_4$  represent?
  - (b) (5 points) If  $w_1 + w_2 + w_3 + w_4 = 18$ , what range of values of  $q$  are needed so that the weighted voting system avoids anarchy and gridlock?
  - (c) (5 points) If  $P_1$  has veto power and  $P_2, P_3,$  and  $P_4$  all vote yes on a motion but  $P_1$  votes no, does the motion pass? Justify your answer.

5. Consider the weighted voting system  $[12 : 9, 6, 4, 2]$ .
- (a) (5 points) Find all possible winning coalitions.
  - (b) (10 points) Find the Banzhaf power distribution.

6. (a) (5 points) What are all possible sequential coalitions in a weighted voting system with 3 players?
- (b) (10 points) Find the Shapley-Shubik power distribution for the weighted voting system  $[4 : 3, 2, 2]$ .

7. (a) (5 points) Calculate  $1 + 2 + \cdots + 35$ .
- (b) (5 points) Calculate  $36 + 37 + \cdots + 100$ .



8. (a) (5 points (bonus)) Let  $A$  and  $B$  be sets. State the definition for  $A$  to be a subset of  $B$ .
- (b) (5 points (bonus)) Give an example of nonempty sets  $A$  and  $B$  such that  $A$  is a subset of  $B$ .