

Math 362: Topics in Geometry

Midterm Exam

Due November 13, 2015

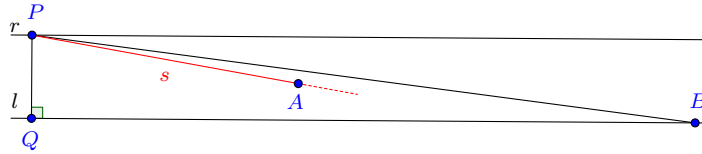
NAME:

To receive full credit you must clearly show all work and justify your answers. Write your final solutions and proofs in this exam packet. If you use GeoGebra or Geometer's Sketchpad for any construction, turn in the electronic file you used to create your construction along with a printout of the construction with your exam. You are allowed to use your notes, GeoGebra, Geometer's Sketchpad, and your book. All constructions must be done **only** with a compass and a straightedge (or equivalent tools with GeoGebra/Sketchpad). **All solutions and proofs must be your own.** This is a 1 week exam that must be turned in no later than **5:00 pm on Friday November 13th.**

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

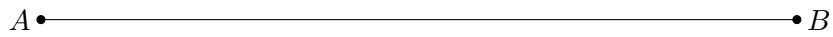
1. (10 points) Without assuming the parallel postulate, prove that the sum of the measure of any two angles in a triangle is less than 180° .

2. (10 points) Prove that the parallel postulate is equivalent to the statement: **The sum of the measures of the angles in any right triangle is 180° .** in the following way.
- (a) Without assuming the parallel postulate, first prove that if the sum of the measures of the angles in any right triangle is 180° , then given a line l and a point P not on l , there exists a unique line through P parallel to l . You may assume that given a fixed value v , there exists a point B on l such that the angle formed at B is less than v . (Hint: Assume for a contradiction and consider two distinct lines r and s through P that are both parallel to l and consider the picture below.)

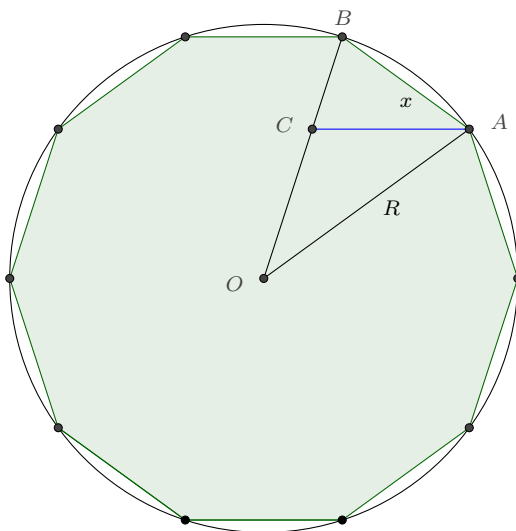


- (b) Assuming the parallel postulate, prove that the sum of the measures of the angles in any right triangle is 180° .

3. (10 points) Using only a straight edge and compass, divide the line segment \overline{AB} (below) into 5 equal segments.



4. (10 points) Let a circle of radius R circumscribe a regular decagon with sides of length x . Prove that $R = x\phi$ where $\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio. (Hint: Consider the picture below where \overline{AC} is the angle bisector of $\angle BAO$. Note that the measure of an exterior angle of a regular decagon is $\frac{360^\circ}{10} = 36^\circ$.)



5. (10 points) Use the result in problem 4 to construct a regular decagon with side length AB from a given segment \overline{AB} .