Introduction to Abstract Mathematics
MATH 310
Homework 8
Due Friday November 6

Please show all your work and justify your answers.

Exercise 1. Use mathematical induction to prove that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for every positive integer n.

Exercise 2. Use mathematical induction to prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for every positive integer n.

Exercise 3. Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n + 1)! - 1$ for every positive integer n.

Exercise 4. Prove that $7 \mid (3^{2n} - 2^n)$ for every nonnegative integer n.

Exercise 5. Prove that if $A_1, A_2, \ldots, A_n$ are any $n \geq 2$ sets, then

$$\overline{A_1 \cap A_2 \cap \cdots \cap A_n} = \overline{A}_1 \cup \overline{A}_2 \cup \cdots \cup \overline{A}_n.$$

Exercise 6. Use the method of minimum counterexample to prove that $3 \mid (2^{2n} - 1)$ for every positive integer n.

Exercise 7. Consider the sequence $F_1, F_2, F_3, \ldots,$ where

$$F_1 = 1,\ F_2 = 1,\ F_3 = 2,\ F_4 = 3,\ F_5 = 5,\ \text{and } F_6 = 8.$$

The terms of this sequence are called **Fibonacci numbers**.

(a) Define the sequence of Fibonacci numbers by means of a recurrence relation.
(b) Prove that $2 \mid F_n$ if and only if $3 \mid n$.

Exercise 8. Use strong induction to prove that for each integer $n \geq 12$, there are nonnegative integers a and b such that $n = 3a + 7b$.