Introduction to Abstract Mathematics
MATH 310
Homework 7
Due Friday 30 October

Please show all your work and justify your answers.

Exercise 1. Give an counter example to the statement: Let $n \in \mathbb{N}$. If $\frac{n(n+1)}{2}$ is odd, then $\frac{(n+1)(n+2)}{2}$ is odd.

Exercise 2. Give a counterexample to the statement: For every positive real number x and every integer $n \geq 2$, the equation $x^n + (x+1)^n = (x+2)^n$ has no integer solutions.

Exercise 3. Prove that the product of an irrational number and a rational number is irrational.

Exercise 4. Prove that $\sqrt{3}$ is irrational. (Hint: first prove for an integer a that $3 \mid a^2$ if and only if $3 \mid a$.)

Exercise 5. Prove that there are infinitely many positive integers n such that $\sqrt{n}$ is irrational. (Hint: consider $\sqrt{2k}$ for any positive integer k.)

Exercise 6. Show that there exists a rational number a and an irrational number b such that a^b is rational.

Exercise 7. Prove that there exist four distinct positive integers such that each integer divides the sum of the remaining integers.

Exercise 8. Disprove the statement: There is a real number x such that $x^6 + x^4 + 1 = 2x^2$.