Introduction to Abstract Mathematics
MATH 310
Homework 6
Due Friday 23 October

Please show all your work and justify your answers.

Exercise 1. Let $a, b \in \mathbb{Z}$, where $a \neq 0$ and $b \neq 0$. Prove that if $a \mid b$ and $b \mid a$, then $a = b$ or $a = -b$.

Exercise 2. Let $x \in \mathbb{Z}$.
(a) Prove that if $2 \mid (x^2 - 5)$, then $4 \mid (x^2 - 5)$.
(b) Give an example of an integer x, such that $2 \mid (x^2 - 5)$, but $8 \nmid (x^2 - 5)$. (Justify your answer.)

Exercise 3. Let $a, b, n \in \mathbb{Z}$, where $n \geq 2$. Prove that if $a \equiv b \pmod{n}$, then $a^2 \equiv b^2 \pmod{n}$.

Exercise 4. Let $m, n \in \mathbb{N}$ such that $m \geq 2$ and $m \mid n$. Prove that if a and b are integers such that $a \equiv b \pmod{n}$, then $a \equiv b \pmod{m}$.

Exercise 5. Prove for every three real numbers x, y and z that

$$|x - z| \leq |x - y| + |y - z|.$$

Exercise 6. Let A and B be sets. Prove that $A \cap B = A$ if and only if $A \subseteq B$.

Exercise 7. Let $A = \{n \in \mathbb{Z} : 2 \mid n\}$ and $B = \{n \in \mathbb{Z} : 4 \mid n\}$ and let $n \in \mathbb{Z}$. Prove that $n \in A \setminus B$ if and only if $n = 2k$ for some odd integer k.

Exercise 8. Let A, B and C be sets. Prove that $(A \setminus B) \cap (A \setminus C) = A \setminus (B \cup C)$.