Introduction to Abstract Mathematics
MATH 310
Homework 3
Due Friday 25 September

Please show all your work and justify your answers.

Exercise 1. Give an example of a statement and determine it's truth value.

Exercise 2. Consider the open sentence $P(x) : x(x-1) = 6$ over the domain $\mathbb{R}$.
(a) For what values of x is $P(x)$ a true statement?
(b) For what values of x is $P(x)$ a false statement?

Exercise 3. For the open sentence $P(A) : A \subseteq \{1, 2, 3\}$ over the domain $S = \mathcal{P}(\{1, 2, 4\})$, determine:
(a) all $A \in S$ for which $P(A)$ is true.
(b) all $A \in S$ for which $P(A)$ is false.
(c) all $A \in S$ for which $A \cap \{1, 2, 3\} = \emptyset$.

Exercise 4. State the negation of each of the following statements.
(a) At least two of my library books are overdue.
(b) One of my two friends misplaced his homework assignment.
(c) No one expected that to happen.
(d) It's not often that my instructor teaches that course.
(e) It's not surprising that two students received the same exam score.

Exercise 5. For the sets $A = \{1, 2, \ldots, 10\}$ and $B = \{2, 4, 6, 9, 12, 25\}$, consider the statements

$$P : A \subseteq B. \ Q : |A \setminus B| = 6.$$

Determine which of the following statements are true.

(1) $P \vee Q$
(2) $P \vee (\sim Q)$
(3) $P \wedge Q$
(4) $(\sim P) \wedge Q$
(5) $(\sim P) \vee (\sim Q)$.

Exercise 6. Let $S = \{1, 2, \ldots, 6\}$ and let $P(A) : A \cap \{2, 4, 6\} = \emptyset.$ and $Q(A) : A \neq \emptyset.$ be open sentences over the domain $\mathcal{P}(S)$.

(1) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \wedge Q(A)$ is true.
(2) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \vee (\sim Q(A))$ is true.
(3) Determine all $A \in \mathcal{P}(S)$ for which $(\sim P(A)) \wedge (\sim Q(A))$ is true.

Exercise 7. For statements P and Q, construct a truth table for $(P \Rightarrow Q) \Rightarrow (\sim P)$.

Exercise 8. Let A and B be nonempty disjoint subsets of a set S. If $x \in S$, then which of the following statements are true?

(1) It's possible that $x \in A \cap B$.
(2) If x is an element of A, then x can't be an element of B.
(3) If x is not an element of A, then x must be an element of B.
(4) It's possible that $x \notin A$ and $x \notin B$.
(5) For each nonempty set C, either $x \in A \cap C$ or $x \in B \cap C$.
(6) For some nonempty set C, both $x \in A \cup C$ and $x \in B \cup C$.

Exercise 9. For open sentences $P(x) : |x - 3| < 1.$ and $Q(x) : x \in (2, 4).$ over the domain $\mathbb{R}$, state the biconditional $P(x) \Leftrightarrow Q(x)$.

Exercise 10. For statements P and Q, the implication $(\sim P) \Rightarrow (\sim Q)$ is called the **inverse** of the implication $P \Rightarrow Q$.

(1) Use a truth table to show that these statements are not logically equivalent.
(2) Find another implication that is logically equivalent to $(\sim P) \Rightarrow (\sim Q)$ and verify your answer.