Introduction to Abstract Mathematics
MATH 310
Homework 1
Due Friday 11 September

Exercise 1. Which of the following are sets?

(a) $1, 2, 3$
(b) $\{1, 2\}, 3$
(c) $\{\{1\}, 2\}, 3$
(d) $\{1, \{2\}, 3\}$
(e) $\{1, 2, a, b\}$.

Exercise 2. Determine the cardinality of each of the following sets:

(a) $A = \{1, 2, 3, 4, 5\}$
(b) $B = \{0, 2, 4, \ldots, 20\}$
(c) $C = \{25, 26, 27, \ldots, 75\}$
(d) $D = \{\{1, 2\}, \{1, 2, 3, 4\}\}$
(e) $E = \{\emptyset\}$
(f) $F = \{2, \{2, 3, 4\}\}$.

Exercise 3. Write each set in the form $\{x \in \mathbb{Z} \mid p(x)\}$, where $p(x)$ is a property concerning x.

(a) $A = \{-1, -2, -3, \ldots\}$
(b) $B = \{-3, -2, \ldots, 3\}$
(c) $C = \{-2, -1, 1, 2\}$.

Exercise 4. The set $E = \{2x \mid x \in \mathbb{Z}\}$ can be described by listing its elements, namely $E = \{\ldots, -4, -2, 0, 2, 4 \ldots\}$. List the elements of the following sets in a similar manner.

(a) $A = \{2x + 1 \mid x \in \mathbb{Z}\}$
(b) $B = \{4n \mid n \in \mathbb{Z}\}$
(c) $C = \{3q + 1 \mid q \in \mathbb{Z}\}$.

Exercise 5. For $A = \{2, 3, 5, 7, 8, 10, 13\}$, let

$$B = \{x \in A \mid x = y + z, \text{ where } y, z \in A\} \text{ and } C = \{r \in B \mid r + s \in B \text{ for some } s \in B\}.$$

Determine C.

Exercise 6. For each of the following give examples of three sets A, B and C such that

(a) $A \subseteq B \subset C$
(b) $A \in B$, $B \in C$, and $A \notin C$
(c) $A \in B$ and $A \subset C$.

Exercise 7. Which of the following sets are equal?

$$A = \{n \in \mathbb{Z} \mid |n| < 2\},\ B = \{n \in \mathbb{Z} \mid n^3 = n\},\ C = \{n \in \mathbb{Z} \mid n^2 \leq n\},$$
$$D = \{n \in \mathbb{Z} \mid n^2 \leq 1\},\ E = \{-1, 0, 1\}.$$

Exercise 8. Find $\mathcal{P}(\mathcal{A})$ and $|\mathcal{P}(A)|$ for $A = \{0, \emptyset, \{\emptyset\}\}$.

Exercise 9. Let $U = \{1, 3, \ldots, 15\}$ be the universal set, $A = \{1, 5, 9, 13\}$, and $B = \{3, 9, 15\}$. Determine the following:

(a) $A \cup B$
(b) $A \cap B$
(c) $A \setminus B$
(d) $B \setminus A$
(e) $\overline{A}$
(f) $A \cap \overline{B}$.

Exercise 10. Give examples of three sets A, B, and C such that $B \neq C$ but $B \setminus A = C \setminus A$.