

# Math 310: Introduction to Abstract Mathematics

## Exam 2

November 11, 2015

<b>NAME:</b>
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To receive full credit you must clearly show all work and justify your answers. No books, notes, or calculators are allowed during this exam. This is a 50 minute exam.

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

- (a) (5 points) Let  $a, b, n \in \mathbb{Z}$  such that  $n \geq 2$ . State the definition of “ $a$  is congruent to  $b$  modulo  $n$ ”.
- (b) (5 points) Is  $37 \equiv 3 \pmod{5}$ ? Justify your answer.

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2. (10 points) Let  $a, b, c, d, n \in \mathbb{Z}$  such that  $n \geq 2$ . Prove that if  $a \equiv b \pmod{n}$  and  $c \equiv d \pmod{n}$ , then  $ac \equiv bd \pmod{n}$ .

3. (10 points) Prove that there exists a number  $a \in \mathbb{R} \setminus \mathbb{Q}$  and a number  $b \in \mathbb{Q}$  such that  $a^b \in \mathbb{R} \setminus \mathbb{Q}$ .

4. (10 points) A sequence  $\{a_n\}_{n \in \mathbb{N}}$  is defined recursively by

$$a_1 = 1, a_2 = 4, \text{ and } a_n = 2a_{n-1} - a_{n-2} + 2 \text{ for } n \geq 3.$$

Prove that for all  $n \in \mathbb{N}$ ,  $a_n = n^2$ .

5. (10 points) Let  $A$  and  $B$  be sets. The **symmetric difference** of  $A$  and  $B$  is defined to be

$$A\Delta B := (A \setminus B) \cup (B \setminus A).$$

Prove that  $A \subseteq B$  if and only if  $A\Delta B \subseteq B$ .