Math 310: Introduction to Abstract Mathematics

Exam 1

October 7, 2015

NAME:

To receive full credit you must clearly show all work and justify your answers. No books, notes, or calculators are allowed during this exam. This is a 50 minute exam.

Question:	1	2	3	4	5	6	Total
Points:	5	10	10	10	5	10	50
Score:							

1. (5 points) Given sets A and B, define $A \cup B$ and $A \setminus B$ using set notation.

2. (10 points) Let $A = \{2, \{3, 5\}\}$, $B = \{\emptyset, \{2\}, 3\}$ and $C = \{\{\emptyset\}, \{3\}, \{\{2\}, 3\}\}$.

   (a) Find $|A|$.

   (b) Is $\emptyset \subseteq A \cap C$?

   (c) Find $|\mathcal{P}(B)|$.

   (d) Find $A \cup C$.

   (e) Is $C \subseteq \mathcal{P}(B)$?

3. (10 points) Let $P(x,y) : x^2+y^2 = 1$ and $Q(x,y) : x+y = 1$ be open sentances over the domain $D = \{(1,-1),(-3,4),(0,-1),(1,0)\}$.

   (a) Determine the truth values for $P(x,y) \Rightarrow Q(x,y)$

   (b) Negate the statement $\forall\,(x,y) \in D, P(x,y) \Rightarrow Q(x,y)$ and determine if it is true or false.

4. (10 points) Let P, Q, and R be statements.

   (a) Complete the following truth table.

P	Q	R	$P \vee Q$	$(P \vee Q) \Rightarrow R$	$P \Rightarrow R$	$Q \Rightarrow R$	$(P \Rightarrow R) \wedge (Q \Rightarrow R)$
T							
T							
T							
T							
F							
F							
F							
F							

   (b) Is $(P \vee Q) \Rightarrow R \equiv (P \Rightarrow R) \wedge (Q \Rightarrow R)$?

5. (5 points) Let A, B, and C be sets such that $A \cap B \cap C \neq \emptyset$. Draw a Venn Diagram that describes the set $(A \cup C) \setminus (A \cap B)$.

6. (10 points) For $r \in I = (0, \infty)$ let $A_r = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y \geq rx^2\}$.

   (a) Sketch $A_0, A_{\frac{1}{2}}$, and A_2.

   (b) Find $\bigcup_{r \in I} A_r$.

   (c) Find $\bigcap_{r \in I} A_r$.