MAT 370 Probability Josh Stangle FALL 2017

TEST 2

Your Name (please PRINT): ____

++++++++++++++++++**INSTRUCTIONS**++++++++++++++++++

- Fill in the above items.
- There is a total of 4 problems, for a maximum possible total value of 40 points. **Make sure you have all 5 test pages (this cover page + 4 test pages)**. You are responsible to check that your test booklet has all 5 pages. Alert a proctor if your copy is missing any pages.
- **Show all your work.** Only minimal credit will be given for answers without supporting work.
- **Write your answer in the box** at the bottom of pages 2-6.
- **Use the back of test pages if additional space is needed**, and for scratch paper.
- **No calculators or other electronic devices; no outside notes; no outside tables** are allowed on this exam. Any use of calculators or electronic devices, or outside notes is a violation of the Academic Integrity Policy.

Do not write below this line

Pb. #	Max Points	Your Score
1	10	
2	10	
3	10	
4	10	
Total	(40)	

1. An etymologist has 500 moths. 300 are male and 200 are female. The moths fly randomly into 100 boxes. The etymologist will open one box at random. Let X and Y be the number of male and female moths in the box, respectively.

(a) Is X best modelled by a uniform distribution, exponential distribution (with parameter $\lambda = 3$) or Poisson distribution (with parameter $\lambda = 3$)?

(b) Using the distribution from part (a), find the probability that the box she opens contains at least 2 male moths. You need not simplify your answer.

(c) Is Y identically distributed to X? Why or why not?

2. Suppose U is a continuous real-valued random variable with uniform density on $[0, 3]$.

(a) Give the density function and cumulative distribution function for U.

(b) Let $Y = e^{U^2}$. Find the density and cumulative distribution function for Y.

(c) Find the probability $Y \leq 2.5$. You need not simplify your answer.

3. In Superior, WI household incomes are normally distributed with mean $\mu = 48000$ and standard deviation $\sigma = 12000$.

(a) Give the density function for the random variable X which is the household income of a randomly selected household in Superior.

(b) In order to join the "Fancy Folks Club" your household needs to make at least \$54000. For a randomly selected household in Superior, find the probability they are able to join this silly club.

4. Suppose X is a random variable with sample space $\{-1, 0, 1, 2\}$. Suppose X has probability distribution m_X with

$$m_X(-1) = .2,\ m_X(0) = .1,\ m_X(1) = C,\ m_X(2) = D,$$

and that $E(X) = 1$.

(a) Find C and D.

(b) Find $V(X)$.

(c) Let $Y = 2X + 1$, find $E(Y)$ and $V(Y)$.

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TEST 2
Take-Home Portion

Your Name (please PRINT): ______________________________

++++++++++++++++++**INSTRUCTIONS**++++++++++++++++++

- Fill in the above items.
- There is a total of 2 problems, for a maximum possible total value of 20 points. **Make sure you have all 3 test pages (this cover page + 2 test pages)**. You are responsible to check that your test booklet has all 3 pages.
- You are not to use any notes or online resources. You may use only our class textbook: *Introduction to Probability* by Grinstead and Snell.
- Please limit yourself to 1.5 hours for this portion of the exam.

Do not write below this line

Pb. #	Max Points	Your Score
1	10	
2	10	
Total	(20)	

1. The time that it takes for a computer system to fail is exponential (in hours) with parameter $\frac{1}{1000}$.

(a) Show that the average (or expected) life expectancy of a computer is 1000 hours.

(b) For the next two parts assume a lab has 10 such computer systems (which fail independently). What is the expected amount of time before the first computer fails? Hint: Problem 7.2.11.

(c) What is the probability that at least two fail before 1000 hours of use? Hint: This is really a question about the number of occurrences in a certain time frame while we know the timing of the occurrences is exponentially distributed (pg. 207).

2. Let X and Y be continuous random variables which are independent and identically distributed with density functions:

$$f_X(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{else.} \end{cases} \qquad f_Y(t) = \begin{cases} 2t, & 0 \le t \le 1 \\ 0, & \text{else.} \end{cases}$$

(a) Give the density function and cumulative distribution function for $Z = X + Y$.

(b) Find the probability that the function $g(x) = x^2 + Zx + 1$ has real roots.