

TEST 1

Your Name (please PRINT): ONLINE VERSION

+++++INSTRUCTIONS+++++

- Fill in the above items.
- There is a total of 5 problems, for a maximum possible total value of 50 points. **Make sure you have all 6 test pages (this cover page + 5 test pages).** You are responsible to check that your test booklet has all 6 pages. Alert a proctor if your copy is missing any pages.
- **Show all your work.** Only minimal credit will be given for answers without supporting work.
- **Write your answer in the box** at the bottom of pages 2-6.
- **Use the back of test pages if additional space is needed,** and for scratch paper.
- **No calculators or other electronic devices; no outside notes; no outside tables** are allowed on this exam. Any use of calculators or electronic devices, or outside notes is a violation of the Academic Integrity Policy.

Do not write below this line

Pb. #	Max Points	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	(50)	

1. You teach a class with 4 students: Jill (female), Omar (male), Jeff (male), and Karen (female). You run the following experiment: You pick one student from the class at random and ask them to pick a color from the set {red, blue, green} at random.

(a) Give the sample space for this experiment explicitly.

(b) Give the event the student is female and does not pick red, explicitly.

(c) Find the probability the color picked is red given that the student is male. Are these events independent? Use the definition of independent to justify your answer.

2. Suppose X is a continuous real-valued random variable with density function

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{x^3}{4} + C, & \text{if } 0 \leq x \leq 2 \\ 0, & \text{if } x \geq 2 \end{cases}$$

(a) Find C so that this is in fact a probability density function.

(b) Find the Cumulative Distribution function for X .

(c) Find the probability the function $g(X) = X^2 + 2X$ is increasing.

3. You find the following Scrabble tiles on your bedroom floor:

$A, A, A, D, F, L, Q, X, Z.$

You choose to waste time thinking about arranging the letters.

(a) How many unique words can be constructed using these tiles? (A word is any string of letters, it need not be identifiable as English.)

(b) If you construct a word at random, what is the probability it begins and ends with A ?

(c) If you construct a word at random, what is the probability it neither starts nor ends with an A ?

4. At a certain university there are 182 calculus I student. Calculus I is taught by 3 different instructors: Professor Amy, Professor Janet, and Professor Cindy. Their classes are different sizes and the grades are as given by the following table.

Table 1: Grades in Calculus

Professor	A	B	C	Total
Amy	18	24	13	55
Janet	21	33	17	71
Cindy	9	28	19	56
Total	48	85	49	182

- (a) What is the probability a student got an A or a B given that Professor Amy was their instructor?
- (b) Use Bayes' Theorem to find the probability a student was in Professor Cindy's class given that they received an A ?
- (c) Are the events that a student received an A and that a student was in Professor Cindy's class independent? Be sure to use the definition of independence to answer.

5. A machine produces batches of 10 light-bulbs at one time. The probability any given light bulb will be defective is 15%.

(a) A batch is rejected if at least 3 light-bulbs are defective. Find the probability a given batch is rejected.

(b) Suppose now instead that the inspector is lazy, and selects just 3 light-bulbs randomly from the batch to test. He rejects the batch if any of these three are defective, otherwise he accepts it. If a batch has 3 defective lightbulbs, find the probability it will be rejected.