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Instructor: Zacharia

MAT 521, Final Exam

Sec 002

NAME:

Instructions: There are 9 problems in this exam for a grand total of 90 points. Each problem is worth about 10 points. Be sure to show your work and explain your reasoning. In general, incomplete or incorrect answers may receive no credit.

1. (12 points) Let X be a random variable with probability density function

$$f(x) = \begin{cases} \dfrac{3}{x^4} & 1 < x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $\Pr(X \leq 2)$.

(b) Find $E(X)$ and $\mathrm{Var}(X)$.

(c) Let $Y = \dfrac{1}{X}$. Find the probability density function of Y. *Be sure your answer is complete!*

2. (10 points) A random variable X has distribution function $F(x) = 1 - e^{-x/5}$ for $-\infty < x < \infty$. (Do not simplify your answer.) Find $\Pr(5 < X < 10)$ and also the p.d.f. of X.

3. (8 points) A random variable X has mean 5 and variance 16. Find numbers a and b such that Y has mean 0 and variance 1, where $Y = \dfrac{X - a}{b}$.

4. (10 points) There are 20 rotted apples in a crate containing 100. Someone picks at random a sample of 20 and without replacement. What is the probability that exactly 5 are rotted? I want the answer **as a decimal**. Explain which distribution you are using.

5. (10 points) A box has 4 cards. Two cards are red on both sides, one card is green on both sides and one card is red on one side and green on the other. A card is selected at random and the color on one side is observed. If this side is red, what is the probability that the other side is also red?

6. (10 points) The probability that a hunter hits his target on any one shot is $1/4$, and they tell us that the shots are all independent of each other. Right. Amazingly, he takes 1,200 shots.
(a) Find the mean and the variance of the number of hits.
(b) Calculate the probability that the number of hits will be at least 330. You should give your answer **as a decimal.**(Hint: one of the tables could prove handy.)

7. (12 points) Let X, Y be random variables with joint p.d.f.

$$f(x,y) = \begin{cases} 6x & \text{if } x, y \geq 0 \text{ and } x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

You may assume (without deriving it) that the marginal p.d.f. of Y is given by the function

$$f_Y(y) = \begin{cases} 3(1-y)^2 & \text{if } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal p.d.f. of the random variable X.
(b) Write the expected value of XY as an explicit integral. (**Do not evaluate this integral.**)
(c) Find $\Pr(X < \frac{1}{3} | Y = 0)$.

8: (8 points) Suppose that A and B are two events in a sample space such that $\Pr(A) = 0.4$, and that $\Pr(B) = 0.1$. Find $\Pr(A \cup B)$ and $\Pr(A^c \cup B^c)$ given that
(a) A and B are disjoint.
(b) A and B are independent.

9. (10 points) Let X and Y be two independent random variables where X has a normal distribution with mean 3 and variance 3, and Y also has a normal distribution, but with mean 5 and variance 4. Compute $\Pr(0 < 2X - Y < 4)$.

You may use this page as scrap paper. Good luck and have a great summer!