

MAT 521
Spring 2001
FINAL EXAM

Show your work. Check your answers. Be sure to identify clearly any events or random variables used in the solutions of the more complex problems. The number in parentheses by each question is its point value out of 200 points.

1. (21) Tom and Sue are members of a hiking club consisting of 14 men and 10 women. The club is about to select a 6 member committee to plan their next hike.

(a) If this committee of 6 is selected at random, what is the probability that it contains 4 men and 2 women?

(b) In how many ways could this 6 member committee be selected so that men and women are equally represented on it?

(c) If 3 men and 3 women are chosen at random for this committee, what is the probability that Tom and Sue are both on the committee?

2. (28) The continuous random variable X has probability density function (p.d.f.)

$$f(x) = \begin{cases} cx^3 & \text{for } 0 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the constant c .

(b) Find $P(0 \leq X < 1)$.

(c) Find $E(X)$.

(d) Find $\text{Var}(X)$.

3. (14) The time required to handle a customer in the checkout line at the certain store is exponentially distributed with a mean time of 5 minutes.
- (a) What is the probability that the customer ahead of you in the line will require at least 3 minutes to check out?
- (b) If three minutes have passed, and the customer ahead of you is still in the process of checking out, what is the probability that at least 3 more minutes will be required for that customer to finish checking out?
4. (9) A jar contains 3 coins. Two of them are ordinary coins with one side heads and the other tails. One of them is a strange two-headed coin. You select a coin from the jar at random and observe that one side of it is heads. What is the probability that the other side is heads?

5. (10) You roll two fair dice, one green and one red. Let the random variable X be the number on the red die minus the number on the green die. Thus, if you roll a 6 on the red die and a 1 on the green die, then X is 5. Similarly, if you roll a 2 on the red die and a 5 on the green die, then X is -3 . If you roll a 4 on both dice, then X is 0. Find the probability mass function (p.m.f.) of X . Display your answer as a table.
6. (10) Mary has just had three job interviews. The first went very well; the probability of getting an offer from that company is 0.9. The second interview went reasonably well; the probability of getting an offer from that company is 0.7. The third interview went rather badly, and the probability of getting an offer from that company is only 0.4. If the decisions of the three companies on making job offers are entirely independent, what is the probability that Mary gets exactly one job offer?

7. (28) Let X and Y be discrete random variables with joint probability mass function (p.m.f.)

X	Y	1	2	3
1		0.10	0.05	0.15
2		0.05	0.10	0.10
3		0.15	0.20	0.10

(a) Find $P(X > Y)$.

(b) Find the marginal probability mass functions of X and of Y . Display your answers below in two tables.

(c) Find $E(X)$ and $\text{Var}(X)$.

(d) Find $P(X > 1 | Y = 1)$.

8. (16) Jane needs to use the university computer system tonight to finish a major project. Unfortunately, the system has just crashed. Seventy percent of the crashes of this system are the result of a software problem. If a crash is the result of a software problem, then the probability that the system will be back up within one hour is 0.8. Twenty five percent of the crashes of the system are caused by a problem with one of the peripherals on the system. Usually the operators can isolate this type of problem and get the system back up. For crashes caused by a problem with the peripherals, the probability that the system will be back up within one hour is 0.6. Five percent of the crashes of this system are the result of a hardware problem with an essential part of the computer. When a crash of this sort occurs, the system is never back up in less than 5 hours.

(a) What is the probability that the system will be back up in an hour so that Jane can finish her project?

(b) One hour later, Jane checks and the system is not up. What is the probability that the crash was caused by a software problem?

9. (10) A local pumpkin grower claims that the weights of the pumpkins he sells are normally distributed with a mean weight of 12 pounds and a standard deviation of 4 pounds. If 25 pumpkins are selected at random from his inventory, what is the probability that their average weight is less than 11 pounds?
10. (14) The scores on a test evaluating the mathematical skills of students are normally distributed with a mean of 70 and standard deviation of 20. The scores on another test which evaluates the verbal skills of students are normally distributed with a mean of 80 and a standard deviation of 15. Assume that the scores on the two tests are independent of each other.
- (a) What is the probability that a student selected at random has a verbal score that is lower than his/her math score?
- (b) What properties of the normal distribution did you use in answering part (a)?

11. (10) A caterer includes sandwiches as a part of her usual luncheon menu. She has found that 10% of the guests at her luncheons do not take any sandwiches, 60% take one sandwich, and 30% take 2 sandwiches. Tomorrow, she is catering a luncheon for 100 people. What is the probability that 132 sandwiches will be enough to provide the guests with as many sandwiches as they want? Assume that the guests' decisions on how many sandwiches they want are made independently.

12. (20) Jim claims that he can taste the difference between cookies made with real butter and cookies made with margarine. His friend Sue doesn't really believe this, and they decide to have a taste test to settle this matter. Sue bakes two batches of cookies which differ only in that one batch is made with butter and the other is made with margarine. She then selects a total of 8 cookies at random from the batches, keeping track of which batch each of the cookies came from. Jim eats these 8 cookies and decides which batch each of them came from. In answering the questions below about this taste test, assume that Jim's success or failure in deciding correctly on each cookie is independent of his success or failure on all the others.

(a) If Jim actually can't tell the difference (so that the probability of correctly guessing which batch a cookie came from is 0.5), what is the probability that he guesses correctly on more than 5 cookies?

(b) Suppose instead that, for each cookie, the probability that Jim decides correctly is 0.7. What is the probability that he decides correctly on no more than 3 cookies?

(c) What distribution did you use to answer parts (a) and (b)?

(d) Give the mean and variance of the distribution used to answer part (b).

13. (10) Tom needs to order a replacement part for his computer system. He can order it from either of two suppliers. All that he knows about supplier A is that the mean delivery time for parts from that supplier is 4 days. For supplier B, he knows that the mean delivery time for parts is 6 days, but he also knows that the standard deviation of the delivery time for parts from supplier B is 2 days. It is very important that he receive this part within 9 days. Given the very little that he knows about the two suppliers, for which supplier can he be the most confident that the part will be delivered within the 9 days? Justify your answer. (Hint: For each supplier, approximate as closely as the available data permits the probability of the delivery requiring 10 or more days.)