Example Identify the Traces and sketch the quadric surface

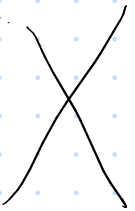
$$\frac{x^2}{2} - \frac{y^2}{4} + z^2 = 1$$

x-traces

x=k $-\frac{y^2}{4} + z^2 = 1 - \frac{k^2}{2} \iff z^2 - \frac{y^2}{4} = 1 - \frac{k^2}{2}$

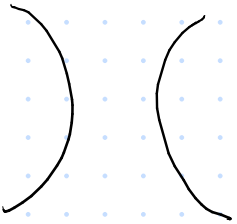
when $k = \sqrt{2}$ $\quad z^2 - \frac{y^2}{4} = 0 \quad z^2 = \frac{y^2}{4}$

$y = \pm 2z$ crossing lines.



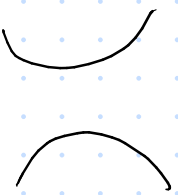
when $k < \sqrt{2}$ $\quad z^2 - \frac{y^2}{4} = C \quad C > 0$

hyperbolas



when $k > \sqrt{2}$ $\quad \frac{y^2}{4} - z^2 = C \quad C > 0$

hyperbolas



$$\frac{x^2}{2} - \frac{y^2}{4} + z^2 = 1$$

y-traces $y = k$

$$\frac{x^2}{2} + z^2 = 1 + \frac{k^2}{4}$$ ellipses.

increase in "diameter" as $k \to \pm\infty$, smallest one is $k=0$.

z-traces $z = k$

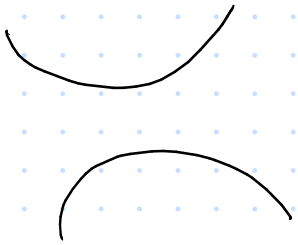
$$\frac{x^2}{2} - \frac{y^2}{4} = 1 - k^2$$

when $k = \pm 1$ $\quad y^2 = 2x^2 \quad y = \pm\sqrt{2}\,x$

crossing lines.

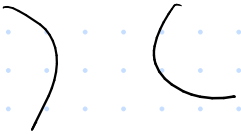
when $|k| < 1$ $\quad \frac{x^2}{2} - y^2/4 = C \quad C > 0$

hyperbolas



when $|k|<1$ $\frac{y^2}{4} - \frac{x^2}{2} = c \quad c>0$

hyperbolas



This is a hyperboloid of one sheet