MAT 242 Calculus III Josh Stangle Spring 2018

TEST 1

Your Name (please PRINT): ______________________________

==================**INSTRUCTIONS**====================

- Fill in the above items.
- There is a total of 5 problems, for a maximum possible total value of 60 points. **Make sure you have all 6 test pages (this cover page + 5 test pages)**. You are responsible to check that your test booklet has all 6 pages. Alert a proctor if your copy is missing any pages.
- **Show all your work.** Only minimal credit will be given for answers without supporting work.
- **Write your answer in the box** at the bottom of pages 2-6.
- **Use the back of test pages if additional space is needed**, and for scratch paper.
- No calculators are allowed on this exam.

Do not write below this line

Pb. #	Max Points	Your Score
1	12	
2	10	
3	10	
4	10	
5	10	
Total	(52)	

1. (a) Let $\vec{b} = \langle 1, 2, 3\rangle$ and $\vec{a} = \langle -2, -1, 4\rangle$. Find the vector projection of $\vec{a}$ onto $\vec{b}$, i.e., find $\text{proj}_{\vec{b}}\vec{a}$

(b) Find a unit vector orthogonal to both $\vec{a}$ and $\vec{b}$ from part (a).

(c) Determine if the 3 points $(1, 0, 1)$, $(1, 2, 2)$, and $(0, 2, 0)$ lie on a plane which also contains the origin.

2. (a) Find the cosine of the angle between the intersecting lines given by vector equations

$$\vec{r}_1(t) = \langle 2 - t, t + 1, 3 - 4t \rangle$$
$$\vec{r}_2(t) = \langle t + 1, 2 - 3t, -(t + 1) \rangle$$

(b) Find the symmetric equations of the line which contains the point of intersection of the two lines above and is orthogonal to both. Hint: Try $t = 1$ in one line and $t = 0$ in the other to find the point of intersection.

3. Find the equation of the plane containing the lines given by vector equations

$$\vec{r}_1(t) = \langle 3, 1, 2 \rangle + t\langle 2, 1, 1 \rangle$$
$$\vec{r}_2(t) = t\langle 4, 2, 2 \rangle$$

4. Let C be the space curve given by $\vec{r}(t) = \langle 2\sin(t), e^t + 2t, \sqrt{t+1}\rangle$. Find the vector equation of the tangent line to the curve at the point given by $t = 0$.

5. Do (a) or (b):

(a) Find the curvature of the twisted cubic $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$.

(b) Prove that a circle of radius a has curvature $1/a$ at any point on the circle.
(Remark: "Because it is on the review sheet" is NOT a sufficient answer.)